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It may be shown that, even when a Fabry–Pérot interferometer is used with plane waves propagating at normal incidence, the variations of the intensity reflected by it with respect to the phase difference (induced by the distance between the two mirrors) are generally not symmetrical around its extrema. We study this problem and express the necessary and general conditions for obtaining a symmetrical optical response in the reflection mode. We analyze the simple case of a Fabry–Pérot interferometer the first mirror of which is constituted by a thin layer of metal. © 2000 Optical Society of America [S0740-3232(00)00101-0]

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1. INTRODUCTION

It is well known that the reflection properties of multilayer stacks are very important in physics and engineering. For many years, we have studied these properties by using the composition law $\oplus$, which is similar to the composition law of velocities in special relativity. Our aim now is to use this composition law $\oplus$ to look at the effect of absorbing media in stratified planar structures on reflection properties. It appears that, for the simple case of a metallized Fabry–Pérot interferometer (to be referred to as a Fabry–Pérot), the reflectivity may be quite asymmetrical even when normal incidence is used (see, for example, Fig. 1 below). As far as we know, this fact was first pointed out by Bruce and Clothier[1] and studied by Monzon *et al.*[2,3] for a system constituted by four interfaces, but to our knowledge it has never been studied systematically. In this paper we discuss the general conditions for having a symmetrical response of the intensity distributions of the reflected and transmitted patterns. In Section 2 we present our notation. We then show in Section 3 that (i) the symmetric or asymmetric response of the reflected intensity of such a device is due to properties of the first mirror only and that (ii) the variation of the transmitted intensity as a function of phase difference is always symmetrical. To go more deeply into the problem, in Section 4 we consider the simpler case of a Fabry–Pérot interferometer the first mirror of which is constituted by a thin metallic layer and the second one by a perfect metallic mirror (i.e., $R_2 = -1$). In Section 5 we present our conclusions.

2. NOTATION

The theory of reflection by multilayer stacks can be found in many books (see, for example, Refs. 4–6). The calculations presented in this paper are based on the matrix representation.[7] To avoid any problem with previous notation, we shall first introduce the physical variables that we use (for more details see Refs. 8–11). The system is constituted by N interfaces separating $N + 1$ media. The jth interface corresponds to the separation of the jth medium and the $j + 1$st medium. For each jth interface the reflection coefficient $r_{j,\,j+1}$ and the transmission coefficient $t_{j,\,j+1}$ for the electrical field propagating in the positive direction are given by the Fresnel coefficients. They depend on the polarization state of the incident light (to simplify our notation, the appropriate subscripts p and s corresponding to the polarization have been dropped from all equations; they could be easily restored). It is important to point out that these coefficients are always defined for a wave traveling in the positive direction, as implied by the order of the subscripts j, $j + 1$. We introduce the complex coefficients of reflection, $R_{j,\,j+1} = r_{j,\,j+1} \exp[2i(\beta_1 + ... + \beta_j)]$ and $\bar{R}_{j,\,j+1} = r_{j,\,j+1} \times \exp[-2i(\beta_1 + ... + \beta_j)]$, and the complex coefficients of transmission, $T_{j,\,j+1} = t_{j,\,j+1} \exp(i\beta_j)$. In these expressions β_j is the propagation phase for a wave traveling in the jth medium (see Ref. 8). The $R_{j,\,j+1}$ coefficient consists of the product of the local Fresnel coefficient of reflection $r_{j,\,j+1}$ of the electric field by the phase $\exp(i\Psi) = \exp[2i(\beta_1 + ... + \beta_j)]$ needed to propagate the electric field from the origin to the jth interface. On the other hand, $\bar{R}_{j,\,j+1}$ consists of the product of the same Fresnel coefficient by the inverse of the phase $\exp(i\Psi)$. We emphasize that consequently, the bar operation does not correspond to the complex conjugation since neither the coefficient $r_{j,\,j+1}$ nor the variables $\beta_1, \ldots, \beta_j$, which can be complex, are affected by this operation.

With these definitions, it can be shown (by using, for example, Ref. 8) that the overall reflection and transmis-

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sion coefficients of a stack made of N interfaces (i.e., $N + 1$ media) is given by

$$\begin{bmatrix} R_{1,N+1} \\ T_{1,N+1} \end{bmatrix} = \begin{bmatrix} R_{1,2} \\ T_{1,2} \end{bmatrix} \otimes \left\{ \begin{bmatrix} R_{2,3} \\ T_{2,3} \end{bmatrix} \otimes \left\{ \cdots \otimes \left\{ \begin{bmatrix} R_{N-1,N} \\ T_{N-1,N} \end{bmatrix} \otimes \begin{bmatrix} R_{N,N+1} \\ T_{N,N+1} \end{bmatrix} \right\} \cdots \right\} \right\}, \qquad (1)$$

where the operation $\otimes$ corresponds to

$$\begin{bmatrix} R_{x,x+1} \\ T_{x,x+1} \end{bmatrix} \otimes \begin{bmatrix} R_{y,y+1} \\ T_{y,y+1} \end{bmatrix} = \begin{bmatrix} \dfrac{R_{x,x+1} + R_{y,y+1}}{1 + \bar{R}_{x,x+1} R_{y,y+1}} \\ \dfrac{T_{x,x+1} T_{y,y+1}}{1 + \bar{R}_{x,x+1} R_{y,y+1}} \end{bmatrix}, \qquad (2)$$

and where x and y are two independent indices. Note that the law $\otimes$ is neither associative nor commutative but appears to be weakly associative and weakly commutative only. This result means that, as shown in Ref. 11, the composition law is associative or commutative apart from a multiplicative factor (note that this multiplicative factor appears to be a phase when neither total reflection nor absorption occurs).

3. GENERAL CASE

We now use this formalism to study the case of a Fabry–Pérot interferometer. Figure 1 shows the variation of the reflectivity of a Fabry–Pérot interferometer with respect to the thickness of the plane-parallel plate of air separating its two mirrors. This figure shows that the reflectivity does not always provide a symmetrical variation around its extrema. Our aim is to look for the general conditions needed to obtain a symmetrical variation of the reflected and of the transmitted intensity around one extremum when the distance between the two mirrors is changed. For simplicity, let us suppose that the two mirrors are separated by vacuum or by a nonabsorbing medium and that they are made of a stack of layers as shown in Fig. 2. Interfaces 1 to M form the first mirror, and interfaces $M + 1$ to N form the second mirror. In this section we do not make any assumption about the material used to build the mirrors, which may be indifferently dielectric or metallic. As shown in Appendix A, Eq. (1) can be written as

$$\begin{bmatrix} R_{1,N+1} \\ T_{1,N+1} \end{bmatrix} = \begin{bmatrix} R_{1,M+1} \\ T_{1,M+1} \end{bmatrix} \otimes \begin{bmatrix} R_{M+1,N+1}\Theta_{1,M+1} \\ T_{M+1,N+1} \end{bmatrix}. \qquad (3)$$

The factor $\Theta_{1,M+1}$ comes from the weak associativity of the law $\otimes$ (see Ref. 9) and from the fact that we have rearranged all the terms into two parts. It shows that the reflected and transmitted amplitudes of the Fabry–Pérot are not obtained from the direct composition of $R_{1,M+1}$ and $T_{1,M+1}$ with $R_{M+1,N+1}$ and $T_{M+1,N+1}$, but they make a factor $\Theta_{1,M+1}$ appear. The general expression of the $\Theta_{1,M+1}$ term is given in Appendix A. By using Eq. (2) we can write the expression of the reflectivity of the overall structure:

$$R = |R_{1,N+1}|^2 = \left| \frac{R_{1,M+1} + R_{M+1,N+1}\Theta_{1,M+1}}{1 + \bar{R}_{1,M+1} R_{M+1,N+1}\Theta_{1,M+1}} \right|^2. \qquad (4)$$

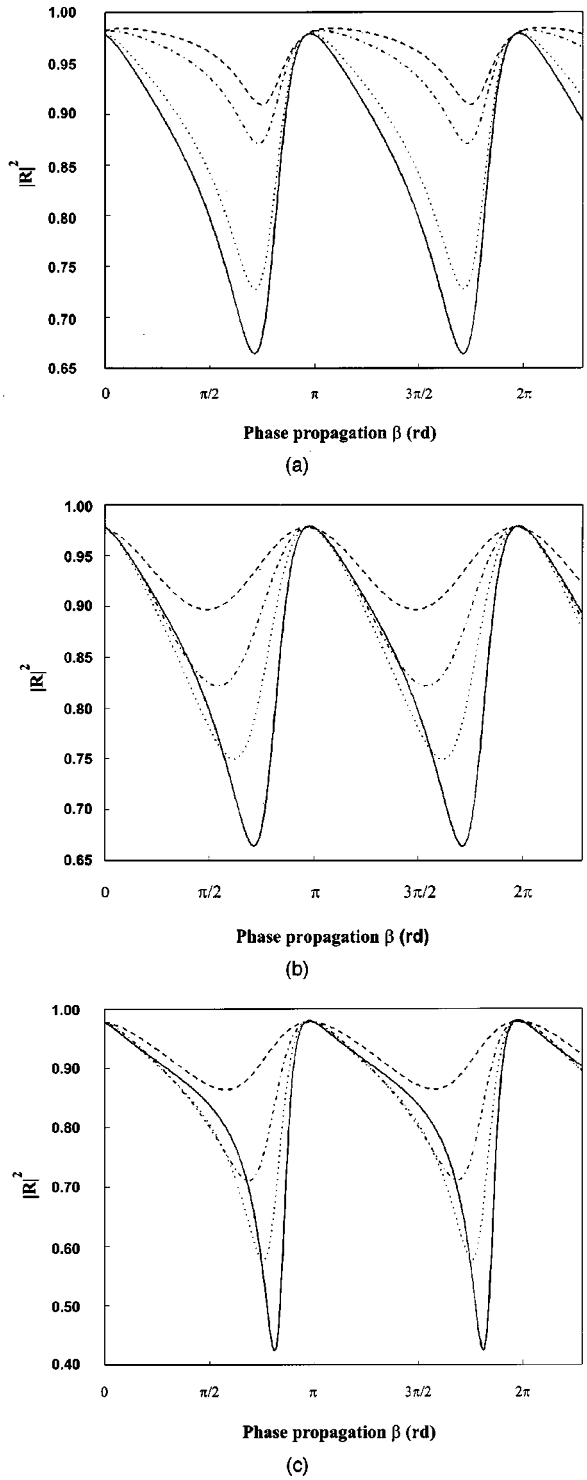


Fig. 1. Study of the reflectivity of the Fabry–Pérot $|R(\beta)|^2$ with respect to the distance between the two mirrors. We denote d as the thickness of the first metallic mirror. (a) $n^I = 8.4$, $\lambda = 1.4\,\mu$m, $d = 3$ nm. Dashed curve, $n^R = 0.05$; dashed–dotted curve, $n^R = 0.1$; dotted curve, $n^R = 0.3$; solid curve, $n^R = 0.4$ (in this range of values of n^R, the curves remain asymmetrical). (b) $n^R = 0.4$, $\lambda = 1.4\,\mu$m, $d = 3$ nm. Dashed curve, $n^I = 2$; dashed–dotted curve, $n^I = 4$; dotted curve, $n^I = 6$; solid curve, $n^I = 8.4$ (the smaller the value of n^I, the more symmetrical the reflectivity). (c) $n = 0.4 + i8.4$, $\lambda = 1.4\,\mu$m. Dashed curve, $d = 1$ nm; dashed–dotted curve, $d = 2.5$ nm; dotted curve, $d = 4$ nm; solid curve, $d = 6$ nm (the smaller the thickness, the more symmetrical the reflectivity).

To simplify our notation, hereafter we write $R_{1,M+1}$ as $\mathcal{R}_1$ (the subscript 1 means the first stack), $R_{M+1,N+1}$ as $\mathcal{R}_2 \exp(-2i\beta)$ (the subscript 2 means the second stack), and $\Theta_{1,M+1}$ as Θ. As noted in Fig. 2, the reflection coefficient of the second mirror can be written as $\mathcal{R}_2 \exp(-2i\beta)$, where β corresponds to β_{M+1}, the propagation of light between the two mirrors; we assume here that β is real. With this notation, Eq. (4) can be expressed by

$$R(\beta) = \left| \frac{\mathcal{R}_1 + \mathcal{R}_2\Theta \exp(-2i\beta)}{1 + \overline{\mathcal{R}}_1\mathcal{R}_2\Theta \exp(-2i\beta)} \right|^2 = \frac{|\mathcal{R}_1|^2 + |\mathcal{R}_2\Theta|^2 + 2|\mathcal{R}_1\mathcal{R}_2\Theta|\cos(2\beta + \varphi_1 - \varphi_2)}{1 + |\overline{\mathcal{R}}_1\mathcal{R}_2\Theta|^2 + 2|\overline{\mathcal{R}}_1\mathcal{R}_2\Theta|\cos(2\beta - \overline{\varphi}_1 - \varphi_2)}. \tag{5}$$

Here, φ_1, $\overline{\varphi}_1$, and φ_2 are the phases of the complex terms $\mathcal{R}_1$, $\overline{\mathcal{R}}_1$, and $\mathcal{R}_2\Theta$. Equation (5) may be written as

$$R(x) = \frac{A + B\cos(x + x_1)}{C + D\cos(x + x_2)}, \tag{6}$$

where *A*, *B*, *C*, and *D* are four real positive numbers and x_1 and x_2 are two real numbers. The positions x_0 of the extrema of $R(x)$ are given by the solution of $\partial R/\partial x = 0$. They verify the following condition:

$$BD\sin(x_2 - x_1) = BC\sin(x_0 + x_1) - AD\sin(x_0 + x_2). \tag{7}$$

The symmetry of the function $R(x)$ about the position x_0 is given by the relation

$$R(x) = R(2x_0 - x). \tag{8}$$

After straightforward calculations, Eqs. (7) and (8) can be combined to yield the condition for a symmetrical response of the Fabry–Pérot in the reflection mode:

$$BD\sin(x_2 - x_1) = 0; \tag{9}$$

that is,

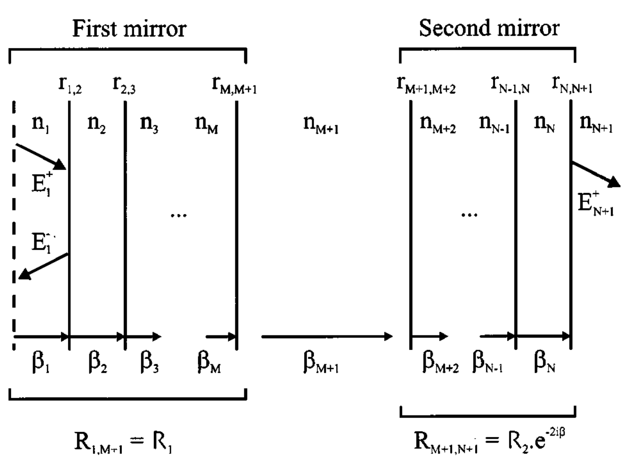


Fig. 2. Definition of our notation in the case of a Fabry–Pérot interferometer. The instrument consists essentially of two stacks made of glass or quartz plates, the inner surfaces of which are coated with partially transparent films of high reflectivity.

$$\begin{cases} B = 0, \\ \text{and/or } D = 0, \\ \text{and/or } x_2 = x_1 + q\pi, \end{cases} \tag{10}$$

where q is an integer. With Eqs. (5) and (6) these conditions can also be expressed as

$$\begin{cases} |\mathcal{R}_1\mathcal{R}_2\Theta| = 0, \\ \text{and/or } |\overline{\mathcal{R}}_1\mathcal{R}_2\Theta| = 0, \\ \text{and/or } \varphi_2 - \varphi_1 = \varphi_2 + \overline{\varphi}_1 + q\pi. \end{cases} \tag{11}$$

The first two conditions are verified only for particular cases obtained with special values of the refractive indices $n_j = n_j^R + in_j^I$ and of β_j. The last one leads to the more general condition,

$$\mathcal{R}_1^* = \mu\overline{\mathcal{R}}_1, \tag{12}$$

where μ is a real number. When the distance between the two mirrors is varied, the reflectivity of a Fabry–Pérot is consequently purely symmetric around its extrema when the complex conjugation of the first reflection coefficient $R_{1,M+1}$ is equal to the same factor barred, apart from a real multiplicative number. We emphasize that Eq. (12) shows that the symmetric or asymmetric character of the reflectivity depends on the physical characteristics of the first mirror only.

Let us now consider the transmissivity of the Fabry–Pérot. Using Eqs. (2) and (3), we can write the transmissivity as

$$T(\beta) = |T_{1,N+1}|^2 = \frac{|T_{1,M+1}|^2 |T_{M+1,N+1}|^2}{|1 + \overline{\mathcal{R}}_1\mathcal{R}_2\Theta \exp(-2i\beta)|^2}. \tag{13}$$

Equation (13) can also be written as

$$T(x) = \frac{E}{C + D\cos(x + x_2)}, \tag{14}$$

where *E*, *C*, and *D* are three real and positive numbers and x_2 is a real number. The positions of the extrema of the function $T(x)$ are given by the solution of

$$ED\sin(x_0 + x_2) = 0. \tag{15}$$

As in Eq. (8), the symmetry of $T(x)$ is expressed by

$$T(x) = T(2x_0 - x), \tag{16}$$

which reduces to the following condition:

$$-2ED\sin(x - x_0)\sin 2(x_0 + x_2) = 0. \tag{17}$$

Taking into account Eq. (15), we see that Eq. (17) is always verified. Consequently, the transmissivity of a Fabry–Pérot is always symmetric about its extrema.

4. STUDY OF A SIMPLE CASE

We now consider a device constituted by two mirrors and illuminated by a plane wave at normal incidence. The first mirror is a single, thin metallic layer a few nanometers thick, and the second one is assumed to be perfect, that is, defined by $\mathcal{R}_2 = -1$ (note that the latter choice does not constitute any restriction of our problem since, as explained in Section 3, the asymmetrical character of the reflectivity depends on the physical characteristics of the first mirror only). Our aim is to study which physical parameters of the first mirror (i.e., its thickness d, the real part n^R, and the imaginary part n^I of its optical index) may influence the asymmetric properties of the Fabry–Pérot.

Figures 1(a) and 1(b) show how the real part and the imaginary part, respectively, of the optical index of the thin layer modify the reflectivity. For a given value of the real part n^R of the metallic layer, the response of the device tends toward a symmetric curve when n^I becomes smaller and smaller [see Fig. 1(b)]. When the imaginary part n^I of the refractive index is taken constant, the observed asymmetry remains approximately the same [see Fig. 1(a)] for the range of values of n^R used in Fig. 1(a). (However, it will be seen in Fig. 5(a) below that for very small values of n^R the response becomes symmetrical again.)

As shown in Fig. 1(c), the same result holds as in Fig. 1(b) when the thickness of the first mirror is changed. The greater the thickness, the greater the asymmetry (this effect, however, can be observed only for small thicknesses because of absorption by the first mirror).

Before studying the condition associated with Eq. (12), for our simple case let us look at conditions $B = 0$ and $D = 0$ [Eqs. (10)]. The condition $B = 0$ can be realized if any of the terms $|\mathcal{R}_2|$, Θ, and $|\mathcal{R}_1|$ is equal to zero. The first case $|\mathcal{R}_2| = 0$ is trivial. The second and third cases lead to the following conditions:

$$r_1 + r_2\rho = 0, \tag{18}$$

$$\rho + r_1 r_2 = 0, \tag{19}$$

where r_1, r_2, and $\rho = \exp(4i\pi d n_{\mathrm{metal}}/\lambda)$ are the reflection coefficient on the first interface (air–metal), the reflection coefficient on the second interface (metal–air) and the phase propagation inside the thin layer, respectively.

The equation $D = 0$ leads to the same condition (19), but condition (18) is changed into

$$r_2 + r_1\rho = 0. \tag{20}$$

The conditions expressed in Eqs. (18), (19), and (20) are special conditions that can be verified in particular cases only. In the present case, $r_1 = -r_2$, and the refractive index of the layer has an imaginary part, so that Eqs. (18) and (20) cannot be verified. In Fig. 3 we study the necessary conditions for having a symmetric curve. As calculated above, this condition is expressed by Eq. (12), in which μ must be a real number. In studying this condition, we see that all figures of that part consequently show the variation of the imaginary part, $\mathrm{Im}(\mu)$, of $\mu = R_1{}^*/\bar{R}_1$ [see Eq. (12)] with respect to the principal characteristics of the medium, that is, n^R, n^I, and the layer thickness d. In these curves, the symmetry is obtained when the imaginary part of μ is zero, that is, when the corresponding curve cuts the x axis.

When studied with respect to the real part n^R of the refractive index [Fig. 3(a)], the curves show that the smaller the value of n^I, the smaller the corresponding value of n^R leading to $\mathrm{Im}(\mu) = 0$. When studied with respect to n^I [Figs. 3(b) and 3(c)], the curves show that the equation $\mathrm{Im}(\mu) = 0$ has no solution for values of n^R smaller than 1

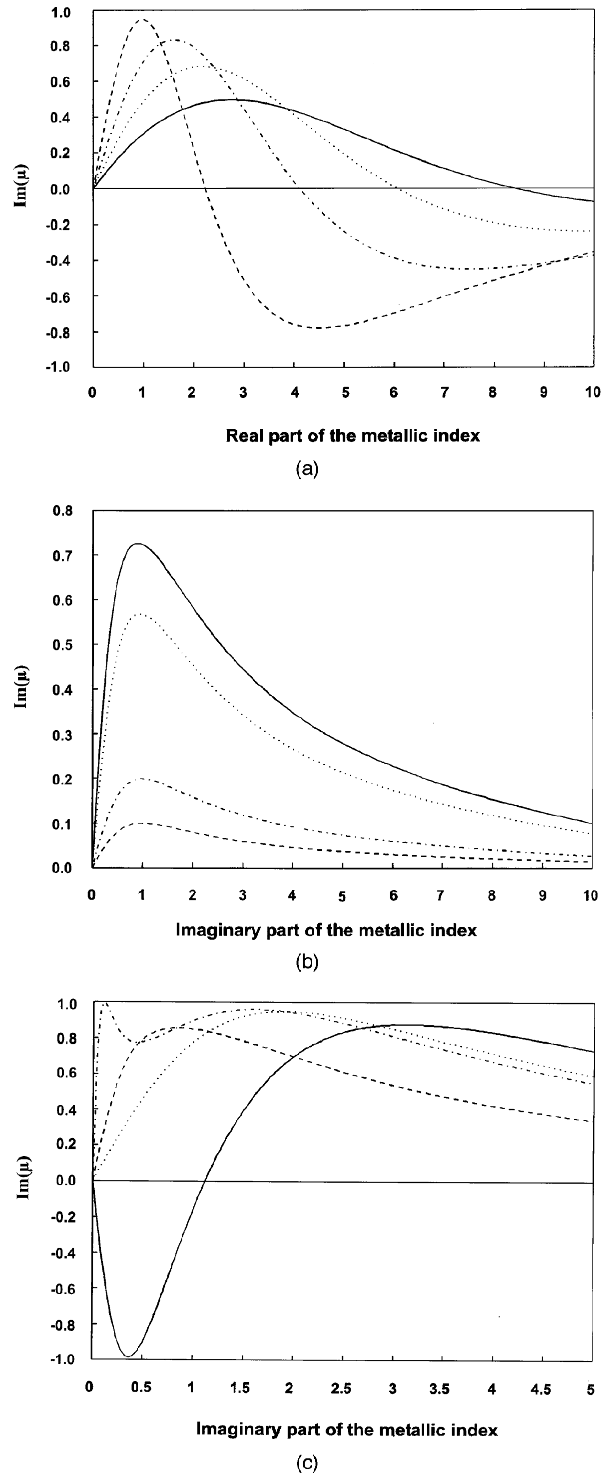


Fig. 3. Conditions for obtaining a symmetrical reflectivity. The reflectivity is symmetrical only when the imaginary part of μ is zero, that is, when the curves cut the x-axis. (a) $\lambda = 1.4$ μm, $d = 3$ nm. Dashed curve, $n^I = 2$, dashed–dotted curve, $n^I = 4$; dotted curve, $n^I = 6$; solid curve, $n^I = 8.4$ (the intersection between the curves and the axis decreases with n^I; it tends toward $+1$ when n^I tends toward 0). (b) $\lambda = 1.4\,\mu$m, $d = 3$ nm. Dashed curve, $n^R = 0.05$; dashed–dotted curve, $n^R = 0.1$; dotted curve, $n^R = 0.3$; solid curve, $n^R = 0.4$. When $n^R < 1$, the curves never cut the axis. (c) $\lambda = 1.4\,\mu$m, $d = 3$ nm. Dashed curve, $n^R = 0.5$; dashed–dotted curve, $n^R = 0.9$; dotted curve, $n^R = 1$; solid curve, $n^R = 1.5$. When $n^R > 1$, the curves cut the axis.

[Fig. 3(b)]. For $n^R > 1$ [Fig. 3(c)], the curves incurvate and can cut the x axis so that a symmetrical response of the device can be obtained.

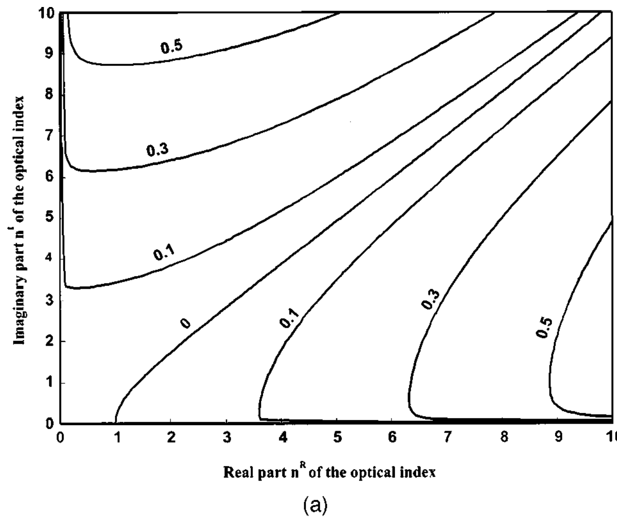
In Fig. 4 we present the position of the symmetrical response of the Fabry–Pérot in the (n^R, n^I) space. Here we use very high values of n^I and n^R that cannot be obtained in experiments. Nevertheless, they are used to show a periodic effect that originates in the phase induced by the propagation of light in the first layer.

In general cases, when the indices are complex, there is little opportunity to get a symmetrical response. Nevertheless, this does not mean that the response could never be symmetrical. These results are illustrated in Fig. 5(a), showing the values of what we call the asymmetry parameter, I_{asym}, in the (n^R, n^I) space. This parameter can be defined as $I_{\text{asym}} = 2|\beta_{\text{mid}} - \beta_{\text{min}}|/\pi$, where β_{mid} is the middle of the distance between two successive maxima where β_{min} is the position of the minimum of $|R(\beta)|^2$. The parameter varies between 0 (for a totally symmetrical response) and 1 (for a totally asymmetrical response). It could easily be shown by presenting some numerical examples that I_{asym} less than 0.05 correspond to such a weak asymmetry that the response would appear symmetrical. For example, in Fig. 1(b) (obtained with $n^R = 0.4$) the four curves corresponding to $n^I = 8.4$, 6, 4, and 2 have an asymmetrical parameter equal to 0.47, 0.29, 0.14, and 0.04 respectively. Moreover, it is important to add that when the refractive index is real or nearly real (and similarly, when it is purely imaginary or nearly purely imaginary) the response always appears to be totally symmetric [as can be seen when one is looking carefully at zones near the axis in Fig. 5(a)].

Figure 5(b) shows the contrast C in the (n^R, n^I) space. The contrast is defined as being

$$C = \frac{|R_{\max}|^2 - |R_{\min}|^2}{|R_{\max}|^2 + |R_{\min}|^2} \tag{21}$$

and varies between 0 and 1, where $R_{\max}$ (respectively, $R_{\min}$) is the maximum (respectively, the minimum) value of $R(\beta)$.



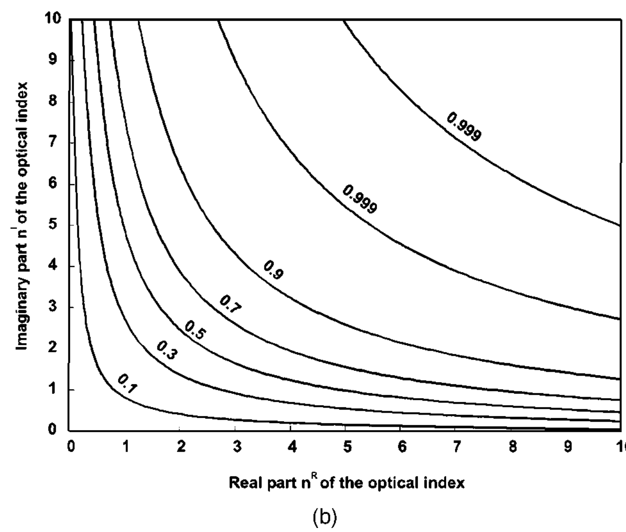


Fig. 5. Variations of the asymmetry parameter and of the contrast in the (n^R, n^I) space when $\lambda = 1.4\,\mu$m, $d = 3$ nm. (a) Representation of the asymmetry parameter I_{asym}. Some contour lines are represented: $I_{\text{asym}} = 0$, totally symmetrical response; $I_{\text{asym}} = 1$, totally asymmetrical response. (b) Representation of the contrast C. Some contour lines are represented: $C = 0$, no contrast; $C = 1$, maximum contrast.

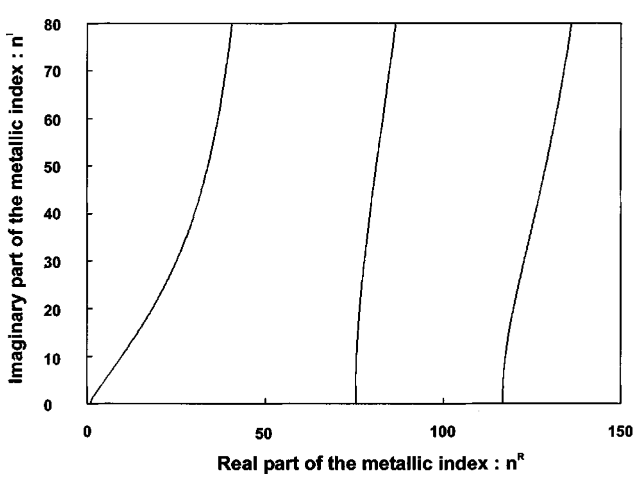


Fig. 4. Representation of the index values corresponding to a symmetrical reflectance curve when $\lambda = 1.4\,\mu$m, $d = 6$ nm. Although they are experimentally impossible, we use here very high values of n^I and of n^R to show the influence of the phase of light inside the first layer.

Figures 5(a) and 5(b) show that the reduction of the dynamic range of reflectivity and the symmetrical characters of the response curves are independent phenomena, so it is possible to get symmetrical curves with a very high contrast (for example, when $n = 6 + i6$ we obtain a symmetrical response with a contrast near 1).

5. CONCLUSION

Our study shows that when used in the reflection mode, the response of a Fabry–Pérot is usually nonsymmetric. This fact has been observed by Bruge and Clothier,[1] but, as far as we know, it has never been systematically studied up to now. It could be used in nonlinear optics to reduce the threshold of a bistable Fabry–Pérot. Note that an important aspect of our study is to show that the phenomenon comes from the physical characteristics of the first mirror only.

APPENDIX A

In this appendix we first recall how the coefficients of reflection and transmission can be calculated by using the formalism introduced by Vigoureux.[8] Then we show the general form of these two coefficients for a Fabry–Pérot interferometer. As is well known, for a multilayer structure, shown in Fig. 2, the coefficients of reflection $R_{1,N+1}$ and of transmission $T_{1,N+1}$ can be expressed by using matrices. Although calculations have been published in Ref. 8, we need some of them here because the notation has been modified.

It was shown in Ref. 8 that the matrix $[M]$ that links the output electric field with the input electric field of the overall structure can be written as

$$[M] = [R_{1,2}][R_{2,3}][R_{N,N+1}][\beta_1 + \beta_2 + \cdots + \beta_N], \quad \text{(A1)}$$

where the matrices $[R_{j,j+1}]$ have the following expression:

$$[R_{j,j+1}] = \frac{1}{t_{j,j+1}}\begin{bmatrix} 1 & \bar{R}_{j,j+1} \\ R_{j,j+1} & 1 \end{bmatrix}. \quad \text{(A2)}$$

It was demonstrated in Ref. 12 that the matrix product $[D_{1,N+1}] = [R_{1,2}][R_{2,3}] \cdots [R_{N,N+1}]$ can be expressed as

$$[D_{1,N+1}] = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} = \frac{1}{\prod_{j=1}^{N} t_{j,j+1}} \begin{bmatrix} \bar{\mathcal{S}}_{2m}^{1,N+1} & \bar{\mathcal{S}}_{2m+1}^{1,N+1} \\ \mathcal{S}_{2m+1}^{1,N+1} & \mathcal{S}_{2m}^{1,N+1} \end{bmatrix}, \quad \text{(A3)}$$

with

$$\mathcal{S}_{2m}^{1,N+1} = \sum_{m\geqslant 0} S_{2m}^{1,N+1},$$

$$\mathcal{S}_{2m+1}^{1,N+1} = \sum_{m\geqslant 0} S_{2m+1}^{1,N+1},$$

$$\bar{\mathcal{S}}_{2m}^{1,N+1} = \sum_{m\geqslant 0} \bar{S}_{2m}^{1,N+1},$$

$$\bar{\mathcal{S}}_{2m+1}^{1,N+1} = \sum_{m\geqslant 0} \bar{S}_{2m+1}^{1,N+1}. \quad \text{(A4)}$$

In Eqs. (A4), the factors $\mathcal{S}_{2m}^{1,N+1}$, $\mathcal{S}_{2m+1}^{1,N+1}$, $\bar{\mathcal{S}}_{2m}^{1,N+1}$, and $\bar{\mathcal{S}}_{2m+1}^{1,N+1}$ can be expressed with the coefficients $R_{j,j+1}$ and $\bar{R}_{j,j+1}$ (Ref. 12). For example, we have

$$S_0^{1,N+1} = 1,$$

$$S_1^{1,N+1} = \sum_{j=1}^{N} R_{j,j+1} = R_{1,2} + R_{2,3} + \cdots + R_{N,N+1},$$

$$\begin{aligned} S_2^{1,N+1} &= \sum_{1\leqslant j<k\leqslant N}^{N} R_{j,j+1}\bar{R}_{k,k+1} \\ &= R_{1,2}\bar{R}_{2,3} + R_{1,2}\bar{R}_{3,4} \\ &\quad + \cdots + R_{1,2}\bar{R}_{N,N+1,} + R_{2,3}\bar{R}_{3,4} + R_{2,3}\bar{R}_{4,5} \\ &\quad + \cdots + R_{2,3}\bar{R}_{N,N+1,} + \cdots + R_{N-1,N}\bar{R}_{N,N+1}, \end{aligned}$$

$$\begin{aligned} S_3^{1,N+1} &= \sum_{1\leqslant j<k<l\leqslant N}^{N} R_{j,j+1}\bar{R}_{k,k+1}R_{l,l+1} \\ &= R_{1,2}\bar{R}_{2,3}R_{3,4} + R_{1,2}\bar{R}_{2,3}R_{4,5} + \cdots \\ &\quad + R_{N-2,N-1}\bar{R}_{N-1,N}R_{N,N+1}, \end{aligned} \quad \text{(A5)}$$

and so on. The bar operation on the coefficients $\bar{\mathcal{S}}_{2m}^{1,N+1}$ and $\bar{\mathcal{S}}_{2m+1}^{1,N+1}$ is done by replacing in $\mathcal{S}_{2m}^{1,N+1}$ and $\mathcal{S}_{2m+1}^{1,N+1}$, respectively, the terms $R_{j,j+1}$ with $\bar{R}_{j,j+1}$ and $\bar{R}_{j,j+1}$ with $R_{j,j+1}$ (note that $\bar{\bar{R}}_{j,j+1} = R_{j,j+1}$), for all the subscripts j,k,l, etc. With our present notation for terms such as $\mathcal{S}_c^{a,b}$, the subscript c indicates the number even (e.g., $2m$) or odd (e.g., $2m + 1$) of terms such as $R_{j,j+1}$ that are factorized in each term of S or $\mathcal{S}$. The superscripts a and b indicate the two limits of the index medium in which the subscript j of the term $R_{j,j+1}$ is taken.

Knowing that the reflection and transmission coefficients of the whole structure are defined by the ratio of E_1^-/E_1^+ and E_{N+1}^+/E_1^+, respectively, and using Eqs. (A3) and (A1), we find that

$$R_{1,N+1} = \frac{E_1^-}{E_1^+} = \frac{M_{21}}{M_{11}} = \frac{D_{21}}{D_{11}} = \frac{\mathcal{S}_{2m+1}^{1,N+1}}{\bar{\mathcal{S}}_{2m}^{1,N+1}}, \quad \text{(A6)}$$

$$\begin{aligned} T_{1,N+1} &= \frac{E_{N+1}^+}{E_1^+} = \frac{1}{M_{11}} = \frac{\exp[i(\beta_1 + \beta_2 + \cdots + \beta_N)]}{D_{11}} \\ &= \frac{\prod_{j=1}^{N} t_{j,j+1}\exp(i\beta_j)}{\bar{\mathcal{S}}_{2m}^{1,N+1}}. \end{aligned} \quad \text{(A7)}$$

Let us now consider that the previous structure is in fact defined by two stacks, the first one limited by the first and the $M + 1$st medium, the second limited by the $M + 1$st and the $N + 1$st medium. Then Eq. (A1) can be written as

$$[M] = [D_{1,M+1}][D_{M+1,N+1}][\beta_1 + \beta_2 + \cdots + \beta_N], \quad \text{(A8)}$$

with

$$[D_{1,M+1}] = [R_{1,2}][R_{2,3}]\cdots[R_{M,M+1}], \quad \text{(A9)}$$

$$[D_{M+1,N+1}] = [R_{M+1,M+2}][R_{M+2,M+3}]\cdots[R_{N,N+1}]. \quad \text{(A10)}$$

Comparing Eqs. (A1) and (A8) and using Eqs. (A3) and (A4), we can write that

$$\mathcal{S}_{2m}^{1,N+1} = \mathcal{S}_{2m}^{1,M+1}\mathcal{S}_{2m}^{M+1,N+1} + \mathcal{S}_{2m+1}^{1,M+1}\bar{\mathcal{S}}_{2m+1}^{M+1,N+1} \quad \text{(A11)}$$

and

$$\mathcal{S}_{2m+1}^{1,N+1} = \mathcal{S}_{2m}^{1,M+1}\mathcal{S}_{2m+1}^{M+1,N+1} + \mathcal{S}_{2m+1}^{1,M+1}\bar{\mathcal{S}}_{2m}^{M+1,N+1}. \quad \text{(A12)}$$

With the use of Eqs. (A11), (A12), (A6), and (A7), and after straightforward calculations, the coefficients $R_{1,N+1}$ and $T_{1,N+1}$ can be expressed as

$$\begin{bmatrix} R_{1,N+1} \\ T_{1,N+1} \end{bmatrix} = \begin{bmatrix} \dfrac{R_{1,M+1} + R_{M+1,N+1}\Theta_{1,M+1}}{1 + \bar{R}_{1,M+1}R_{M+1,N+1}\Theta_{1,M+1}} \\ T_{1,M+1}\dfrac{T_{M+1,N+1}}{1 + \bar{R}_{1,M+1}R_{M+1,N+1}\Theta_{1,M+1}} \end{bmatrix} = \begin{bmatrix} R_{1,M+1} \\ T_{1,M+1} \end{bmatrix} \otimes \begin{bmatrix} R_{M+1,N+1}\Theta_{1,M+1} \\ T_{M+1,N+1} \end{bmatrix}, \tag{A13}$$

where

$$\Theta_{1,M+1} = \frac{\mathcal{S}_{2m}^{1,M+1}}{\bar{\mathcal{S}}_{2m}^{1,M+1}}. \tag{A14}$$

As an example, for three and four media the expressions of $\Theta_{1,3}$ and $\Theta_{1,4}$ are the following:

$$\Theta_{1,3} = \frac{1 + R_{1,2}\bar{R}_{2,3}}{1 + \bar{R}_{1,2}R_{2,3}},$$

$$\Theta_{1,4} = \frac{1 + R_{1,2}\bar{R}_{2,3} + R_{1,2}\bar{R}_{3,4} + R_{2,3}\bar{R}_{3,4}}{1 + \bar{R}_{1,2}R_{2,3} + \bar{R}_{1,2}R_{3,4} + \bar{R}_{2,3}R_{3,4}}. \tag{A15}$$

These terms appear to be equivalent to a general geometric phase (see Ref. 9).

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